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Research Paper

Analysis of incident surface wave characteristics in 3-dimension for a fluid of finite depth

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Abstract. When working on offshore rigid or floating bodies, it is important to understand how these structures respond to waves. Ocean surface waves are known to cause periodic loads on all man-made structures in the sea, whether these structures are fixed, floating or sailing and on the surface or deeper in the sea. To understand these loads, a good understanding of the physics of water waves is necessary. Analysis of incoming waves is especially of great importance to ocean engineers and designers. Furthermore, these waves have essential characteristics, which ought to be properly understood. These include incident wave potential, incident wave elevation, wave velocity and acceleration. The objective of this paper is to analyze vertical wave elevation, velocity and acceleration, as a result of incident wave potential. We also investigate the influence of water depth on the wave characteristics. The incident velocity potential is solved by separation of variables where appropriate boundary conditions are imposed.

Keywords: Incident surface wave velocity potential, wave elevation, velocity and acceleration, floating offshore structures, slamming.

INTRODUCTION

Surface waves are caused by wind, moving vessels, seismic disturbance of shallow sea floors (tsunamis) and the gravitational disturbance of the sun and the moon. These waves differ only because of their periods. Surface wave causes periodic loads on all the man-made structures in the sea regardless of whether these structures are rigid or floating or whether they are deep in the ocean or on the surface (Finnegan et al., 2013). When offshore bodies such as ships move through an incident wave fields, they are exposed to forces due to the presence of oncoming waves and due to their diffraction. This is brought about by the interaction between water waves and floating bodies. Therefore, the prediction and analysis of the sea environment such as surface waves, ship response, wave load, deck wetness, slamming among others are very important aspects in ship design. Surface waves induce motion on ships and other offshore structures (Faltinsen, 2005). These wave-induced motions sometimes reduce the performance and the workability of the structures. This has received considerable attention from designers since they have acknowledged the fact that to design an offshore structure, accurate predictions for the hydrodynamics' loads is very important and the sea environment (Manyanga and Wen-Yang, 2012). These loads are present maybe due to factors such as ground acceleration, ice impact or wave excitation. Coastal engineering is a very active field and researchers in this field are motivated to study the response of offshore structures to incident waves. According to the linear theory, the velocity potential and the fluid potential are constant from the mean surface to the actual surface level (Faltinsen, 2005). This theory is widely used in modeling sea environment and in the derivation of the hydrodynamic waves. It has helped in giving a good description of the wave kinematic and dynamic characteristics and in the analysis of the effects they have on offshore structures especially diffraction problem.

In the present paper, linear theory is used in the derivation of the incident wave potential. Moreover, the analysis of waves by linear theory is important since it assumes that the amplitude of the incident wave is linearly proportional to the wave induced motion amplitude. This means that the motion associated with any offshore structure depends on the wave loads that excite this structure.

Most of the research work available aimed at understanding the waves induced motion on offshore structures. However, this paper focuses on first understanding the waves (incident waves) even before they come into contact with the offshore structures or bodies either if the body is in motion or not. The analysis of the velocity, acceleration and even pressure is quite important since it helps in determining the wave-induced forces on any offshore structure. For example when dealing with diffraction problem, incident waves are used in determining the Froude-Krylow force, hence it is essential to understand incident waves (Vasquez et al, 2001). In this present work, linear theory (Coastal Engineering Research Centre, 1977) is used in the derivation the velocity potential. Consequently, the wave characteristics that is; wave elevation, velocity and acceleration are derived and their result in each spatial axis compared.

This study has been motivated by the ferry accident that took place on 01 May 2014 in South Korea and left 284 people dead and 20 missing. This is despite the sending of 170 rescue ships and 500 divers. The rescue mission was impossible due to whipping winds and heavy currents on the area, which caused very strong surface waves.

MATHEMATICAL FORMULATION

In this paper, we assume that the water is incompressible (constant density), inviscid (no friction) and the motion is irrotational (the curl of the fluid velocity is zero). It is assumed that the water is of finite depth and the seabed is horizontal, fixed and impermeable. A Cartesian system is adopted, $o-xyz$, fixed on the fluid with oy opposing the direction of gravity and $o-xz$ lying on the undisturbed free surface. The plane $y=0$ denotes the undisturbed water surface. The surface displacement of the water from the mean water level is given by $y=\eta\left(x,z,t\right)$ which is the wave elevation. The water depth, h , is measured between the seabed $\left(y=-h\right)$ and the still water level $\left(y=0\right)$. Surface water waves are considered. The amplitude of the wave which is the distance between the still water level and the wave crest is assumed to be small than its wavelength λ . The height is the distance from the crest to the trough so that the wave amplitude is given as $a=\frac{H}{2}$. The wave is assumed to be periodic so that the wave period is taken as the time required by one wave to pass a particular point. This problem is governed by certain boundary conditions.

Governing equations

Since the fluid is incompressible, the divergence of its velocity field is zero everywhere:

$$\nabla \bullet V = 0 \tag{1}$$

The flow is irrotational, then there exist a velocity potential $\phi(x,y,z)$ such that the velocity components in the $x,y,z-axis$ is given as:

$$\frac{\partial\phi}{\partial x}=u,\frac{\partial\phi}{\partial y}=v,\frac{\partial\phi}{\partial z}=w \tag{2}$$

Substituting Equations 1 into 2 the velocity potential satisfies Laplace equation:

$$\nabla^2\phi=0 \tag{3}$$

$$\frac{\partial^2\phi}{\partial u^2}+\frac{\partial^2\phi}{\partial v^2}+\frac{\partial^2\phi}{\partial z^2}=0 \tag{4}$$

The velocity potential is subject to kinematic and dynamic free surface boundary conditions.
The boundary conditions for this case are as follows:

1. The velocity components in the y -direction must go to zero at the seabed.

$$v=\frac{\partial\phi}{\partial y}=0, \text{ at } y=-h \qquad \text{(Faltisen,1990)} \tag{5}$$

2. The pressure at the free surface is equal to the atmospheric pressure.

$$\frac{\partial\phi}{\partial t}+\frac{1}{2}\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2+\left(\frac{\partial\phi}{\partial z}\right)^2\right]+\frac{p}{\rho}+gy=0 \tag{6}$$

Since in this study linear theory is used then the high powers in Equation 6 will be ignored and the equation it reduces to:

$$\left(\frac{\partial\phi}{\partial t}\right)_{y=-h}+g\eta=0 \quad (7)$$

Where $y=\eta\left(x,z,t\right)$ (8)

Let $F\left(x,y,z\right)$ (9)

$$\frac{DF}{Dt}=\left(\frac{\partial F}{\partial t}+u\frac{\partial F}{\partial x}+w\frac{\partial F}{\partial z}-y\right)_{y=0}=0 \quad (10)$$

$$\frac{\partial\eta}{\partial t}+\frac{\partial\eta}{\partial x}\left(\frac{\partial\phi}{\partial x}\right)_{y=\eta}+\frac{\partial\eta}{\partial z}\left(\frac{\partial\phi}{\partial z}\right)_{y=\eta}=\left(\frac{\partial\phi}{\partial y}\right)_{y=\eta} \quad (11)$$

From Equation 7 we have:

$$\left(\frac{\partial\phi}{\partial t}\right)_{y=0}+g\eta=0 \quad (12)$$

And from equation (11) we have:

$$\frac{\partial\eta}{\partial t}=\left(\frac{\partial\phi}{\partial y}\right)_{y=0} \quad (13)$$

From Equations 11 and 12 we have:

$$-\omega^2\phi+g\frac{\partial\phi}{\partial y}\text{ , at y = 0} \quad (14)$$

Solution for the velocity potential

We assume that the free surface wave elevation η take the form:

$$\eta=ae^{i\left(k_1x\cos\theta+k_2z\sin\theta-\omega t\right)} \quad (15)$$

The aim is to find the velocity potential ϕ satisfying the above boundary conditions.

Equation 4 is solved by the separation of variables. We assume that the incident wave velocity potential takes the form:

$$\phi=f(y)e^{i(k_1x\cos\theta+k_2z\sin\theta-\omega t)} \quad (16)$$

$$\nabla^2\phi=\frac{\partial^2 f}{\partial y^2}-k^2f\left(y\right)=0 \quad (17)$$

Where $k=\sqrt{\left(k_1^2+k_2^2\right)}$ (18)

Solving Equation 17 we have:

$$y=Ae^{ky}+Be^{-ky} \quad (19)$$

Putting Equations 19 into 16 we have:

$$\phi=\left(Ae^{ky}+Be^{-ky}\right)\left(e^{i(k_1x\cos\theta+k_2z\sin\theta-\omega t)}\right) \quad (20)$$

But from (4) we have

$$Ae^{-kh}-Be^{kh}=o$$
$$Ae^{-kh}=Be^{kh} \quad (21)$$

But from (13) we have:

$$\left(\omega^2-gk\right)A+\left(\omega^2+gk\right)B=0 \quad (22)$$

From (19) and (20) we have:

$$\begin{pmatrix} e^{-hk} & -e^{hk} \\ \left(\omega^2-gk\right) & \left(\omega^2+gk\right)\end{pmatrix}=0 \quad (23)$$

$$\omega^2=gk\left(\frac{\left(e^{kh}-e^{-kh}\right)}{\left(e^{kh}+e^{-kh}\right)}\right)=gk\tanh kh \quad (24)$$

This is the linear dispersion relation, where k is the wave number and can be regarded as eigen- value. The transcendental equation (24) has only one eigen-value; and hence corresponding to the eigen-value k, there is only one eigen function $Y\left(y\right)$ (Rahman and Bhatta, 1992). From this relation we see that waves of different wavelength travel at different speed and if more than one wave is present then the wave with the longer period travels faster.

$$c=\frac{\omega}{k}$$
$$c=\sqrt{\left(\frac{g}{k}\tanh hk\right)} \quad (25)$$

This equation shows that the wave celerity does not depend on the height of the wave or the on the amplitude. For waves with different period, the one with a longer period propagates at high celerity consequently moves ahead unlike the one with shorter period. This shows that waves generated by seismic disturbances (tsunamis) move ahead of others at very high celerity and their effect are catastrophic as compared to other surface gravity waves.

Let, $Ae^{-kh} = Be^{kh} = \frac{D}{2}$ (26)

Hence (18) becomes,

$$\phi = \frac{1}{2} D \left\{ e^{k(h+y)} + e^{-k(h+y)} \right\} \left(e^{i(kx\cos\theta + kz\sin\theta - \omega t)} \right) \quad (27)$$

$$\phi = \left(D\cosh k(h+y) \right) e^{i(kx\cos\theta + kz\sin\theta - \omega t)} \quad (28)$$

But, $\left(\frac{\partial \phi}{\partial t} \right)_{y=0} = \eta$ (29)

$$\eta = \frac{i\omega}{g} D\cosh kh \left(e^{i(kx\cos\theta + kz\sin\theta - wt)} \right) = -\frac{1}{g} \left(\frac{\partial \phi}{\partial t} \right) \quad (30)$$

$$\eta = ae^{i(kx\cos\theta + ky\sin\theta - \omega t)} \quad (31)$$

Then from (24) we have,

$$\frac{\omega^2}{g} \cosh kh = k\sinh kh \quad (32)$$

$$D = -i\frac{ag}{\omega\cosh kh} = -i\frac{a\omega}{k}\frac{1}{\sinh kh} \quad (33)$$

Hence, $\phi = -i\frac{a\omega}{k}\frac{\cosh k(h+y)}{\sinh kh} e^{i(kx\cos\theta + kz\sin\theta - \omega t)}$ (34)

However, by Euler's formula: $e^{i\alpha t} = \cos\alpha t + i\sin\alpha t$

Hence, Equation 34 becomes:

$$\phi = -i\frac{a\omega}{k}\frac{\cosh k(y+h)}{\sinh kh}\left(\cos(kx\cos\theta + kz\sin\theta - wt) + i\sin(kx\cos\theta + kz\sin\theta - \omega t) \right)$$

$$\phi = \frac{a\omega}{k}\frac{\cosh k(y+h)}{\sinh kh}\sin(kx\cos\theta + kz\sin\theta - \omega t) \quad (35)$$

Equation 35 is the velocity potential satisfying the boundary conditions aforementioned.

Wave elevation

From Equation 35 and the boundary condition on Equation 29 the wave elevation can be derived. Wave elevation is an important aspect of the wave since it is used in the determination of the vertical relative motion of any floating structure with respect to the undisturbed wave surface. Vertical motion is a very significant aspect that coastal engineers should always put into consideration. For instance in ships, vertical motions are used to predict damages that might occur due to slamming and water in the deck.

$$\frac{\partial \phi}{\partial t} = \eta = \frac{a\omega^2 \cosh k(y+h)}{k\sinh kh}\cos(kx\cos\theta + kz\sin\theta - \omega t) \quad (36)$$

This is a cosine wave profile.

Wave velocity

Wave velocity is derived directly from the velocity potential. This is possible due to the irrotationality condition of the fluid.

$$u = \frac{\partial \phi}{\partial x} = \frac{a\omega\cos\theta\cosh k(y+h)}{\sinh kh}\cos(kx\cos\theta + kz\sin\theta - \omega t)$$

$$v = \frac{\partial \phi}{\partial y} = \frac{a\omega\sinh k(y+h)}{\sinh kh}\sin(kx\cos\theta + kz\sin\theta - \omega t) \quad (37)$$

$$w = \frac{\partial \phi}{\partial z} = \frac{aw\sin\theta\cosh k(y+h)}{\sinh kh}\cos(kx\cos\theta + kz\sin\theta - \omega t)$$

These velocity equations express the local fluid velocities at any distance *(y+h)* above the impermeable bottom. Furthermore, they are periodic. The hyperbolic functions give the exponential decay of the magnitude of the velocity components in respect to increase of distance below the free surface.

Wave acceleration

Acceleration is the change of velocity with respect to time. The negative sign shows that the direction of the wave is changing with respect to the origin.

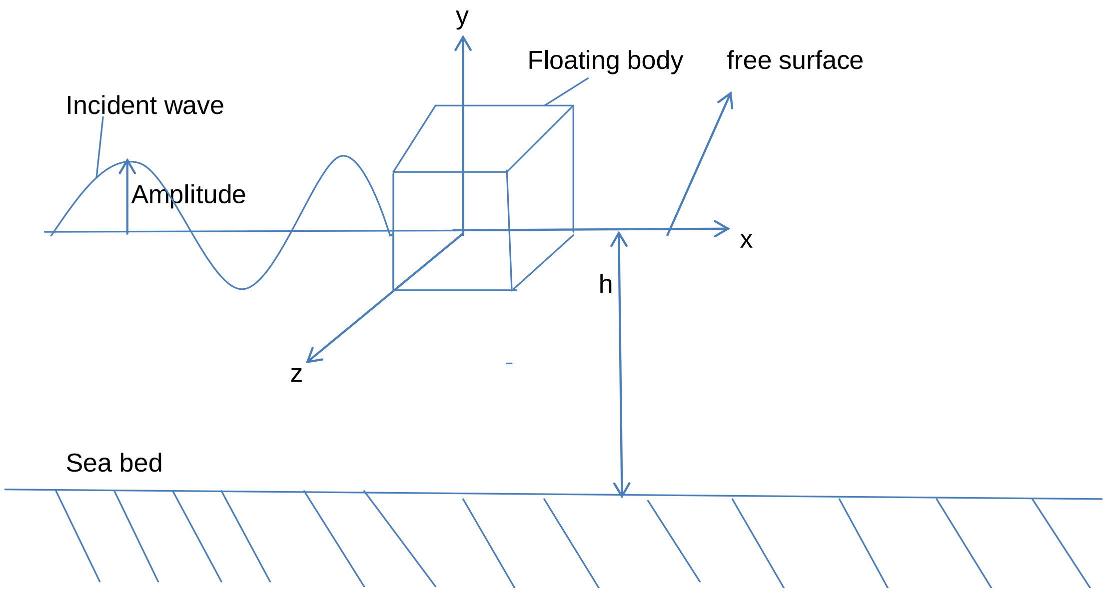

Figure 1. Schematic diagram representing rectangular floating body on the incident wave field.

$$u_t = \frac{a\omega^2 \cos\theta \cosh k(y+h)}{\sinh kh} \sin(kx\cos\theta + kz\sin\theta - \omega t)$$
$$v_t = -\frac{a\omega^2 \cosh k(y+h)}{\sinh kh} \cos(kx\cos\theta + kz\sin\theta - \omega t) \quad (38)$$
$$w_t = \frac{a\omega^2 \sin^2\theta \cosh k(y+h)}{\sinh kh} \sin(kx\cos\theta + kz\sin\theta - \omega t)$$

The acceleration of the wave is linearly proportional to that of the body. Vertical acceleration in particular between the body and the waves is responsible for determining the cargo weight in a ship and has also been associated to seasickness.

RESULTS AND DISCUSSION

Figure.2 shows the relationship between wave elevation, velocity and acceleration in the vertical direction. From the figure it is observed that the wave's vertical velocity and acceleration are out of phase by 90°. The period of this wave remains constant. Manyanga and Duan (2011) also observed this in their study of Three Dimension Internal Waves due to Pulsating Sources and Oscillation of floating Bodies. The amplitude of the wave velocity motion is larger than that of acceleration. Moreover, wave energy is directly proportional to the square of the amplitude of the wave (it is correlated to intensity energy per area per time). Furthermore, from the oscillatory wave structures we observe that when the velocity amplitude is at maximum, the acceleration amplitude is at a minimum.

In conclusion, waves with high vertical velocity will induce a lot of motion on a body especially if the body is so close to the origin of the wave. Very high velocity impacts between offshore structures and the waves are associated with slamming. However, since the impact occurs over a small period of time, the gravity accelerations is assumed negligible relative to the impact induced accelerations (Baarholm and Stansberg, 2005).

When an offshore structure is hit by these waves, the added mass and damping coefficients always correspond to components that are in phase with the accelerations and velocity of the structure. However if the added mass and the damping coefficients of the body is zero then the oncoming waves is equal to the outgoing waves. In summary, acceleration and velocity of the structure is proportional to the acceleration and velocity of the surface waves respectively.

Figure 3 shows the relation between elevation, velocity and acceleration in the direction of wave propagation. On the other hand, comparing Figures 2 and 3, we see that the velocity in x and the elevation have a very big difference. For instance, when wave elevation motion amplitude is at maximum, the velocity amplitude is at minimum, they form a standing wave. This shows that the horizontal force generated by the waves does not induce a lot of motions on the floating body. We conclude that the horizontal characteristic of wave does not lead to high-induced motion.

It is clearly depicted from Figure 4 that the velocity of the wave motion in the z-axis is negligible. The amplitude of the entire wave characteristic as far as this axis is concerned is small in comparison to the other axes.

Figure 5 shows that the depth of the fluid greatly affects

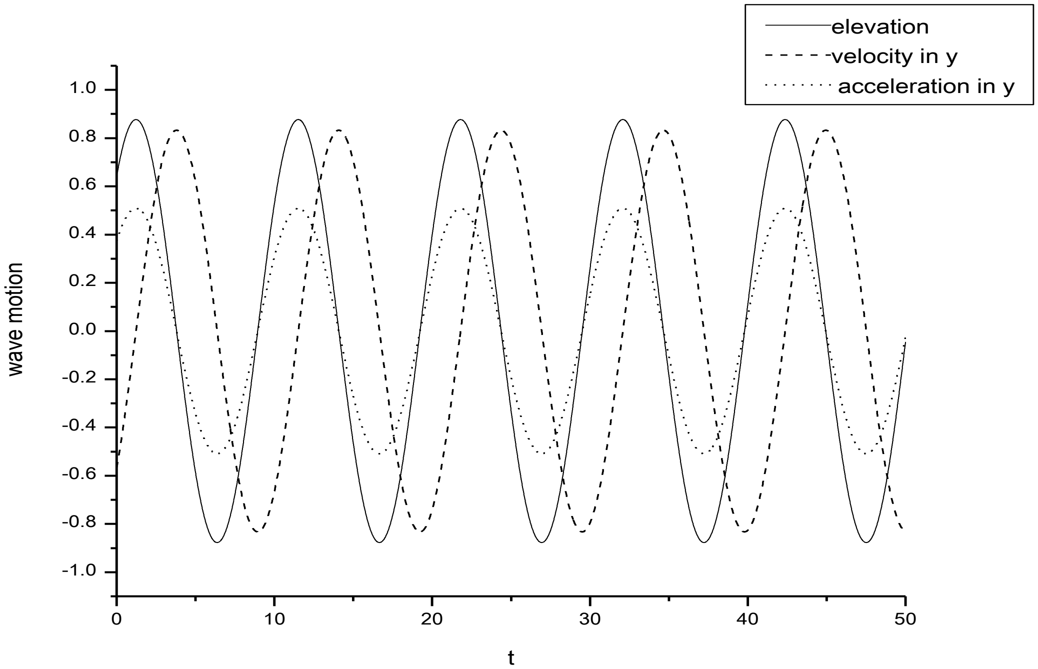

Figure 2. Graph showing the relationship between wave elevation and vertical velocity and acceleration.

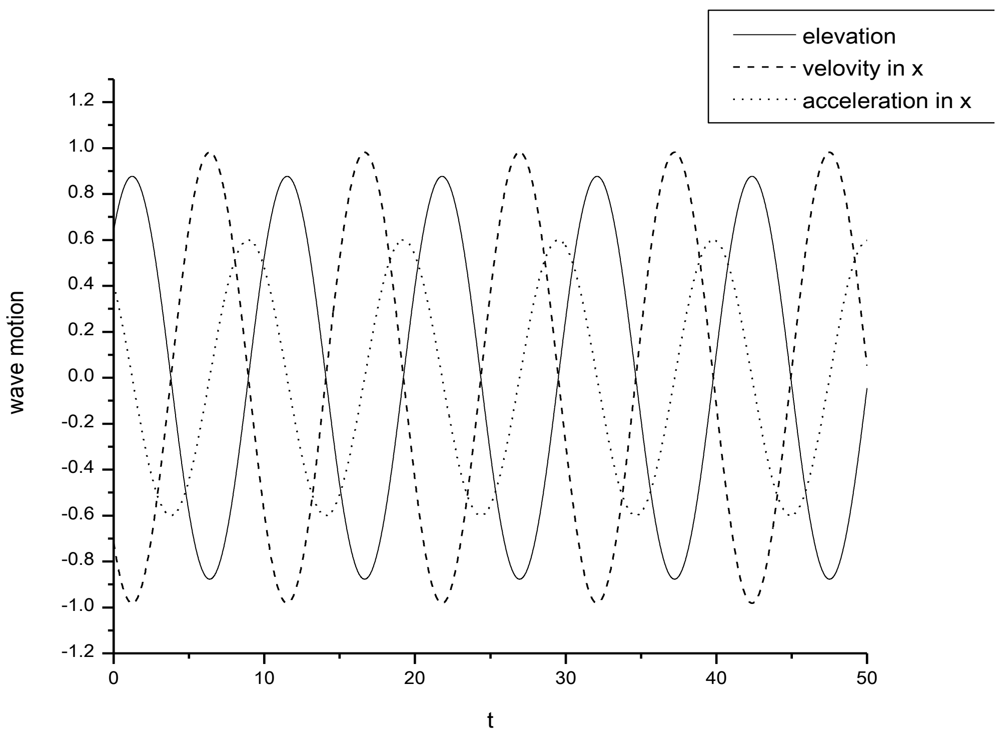

Figure 3. Graph showing the relationship between wave elevation, velocity and acceleration on the x direction.

the number of oscillations. As the depth increases, the frequency also increases. Furthermore, as the depth increases it is clear that the wavelength also increases though at a slower rate. In addition, we see that the horizontal particle displacement is large when the depth is small. In summary, it is observed that most waves'

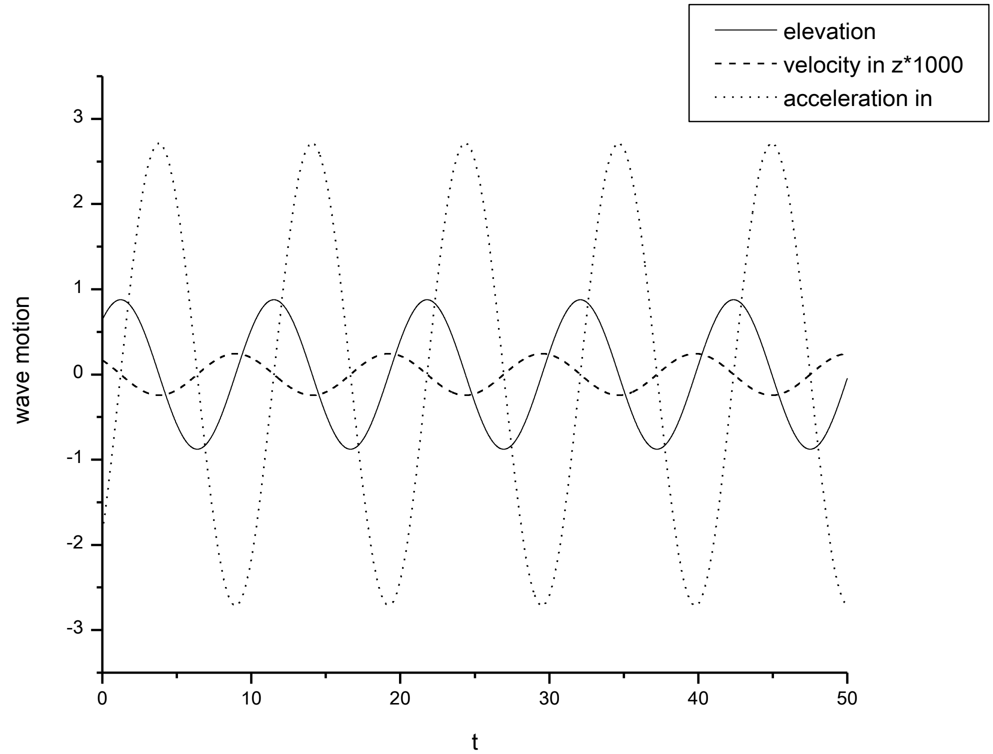

Figure 4. Graph showing the relationship between wave elevation, velocity and acceleration in z direction.

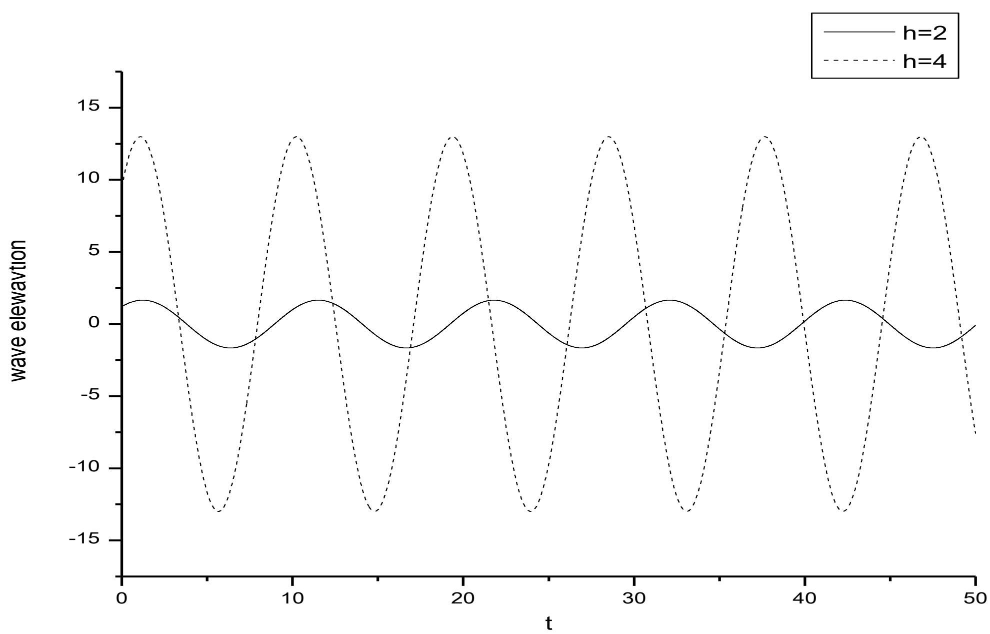

Figure 5. Showing the wave elevation at different water depth from the mean surface.

characteristics that is; celerity, height, length, surface profile, water particles velocity and acceleration are affected by change in depth. However, the wave period remains constant.

CONCLUSION

The decay of waves generated perpendicular to the offshore structure's course has been of great importance to coastal engineer's. For instance, when a wave enters shallow water its wavelength always reduces but the wave amplitude increases. Manyanga et al. (2012) illustrated that the amplitude of the surface waves decreases as the depth increases, which is in accordance to the shallow water theory. For example, a ship navigating in rough seas has a lot of impact on water. This is due to the presence of relative vertical motion (Inoue and Zakaria, 2005). The impact causes slamming, (the impulse loads with high-pressure peaks that occur between the ship and water) whose probability is higher when the vertical velocity between the ship and the wave is large. From the presented analysis, findings show that incident waves have adverse effects on the sea environment and on any floating structures on the wave field. For instance, it is observed that the vertical velocity and vertical acceleration significantly contribute tremendous vertically induced motions. In this sense, it is advisable to evaluate the wave characteristic and modify offshore structures to withstand extreme wave fluctuations. In particular, the ship coxswain is expected to allow it to propel subject to restoring force in order to reduce the wave intensity by utilizing the body's induced motion. These consequently generate waves which exactly cancels the incident waves. This can be done by ensuring that the velocity of the moving body is chosen such that incident waves are totally reflected (Baarholm and Stansberg, 2005). If engineers are able to make a good analysis of the incident waves then a good prediction can be made on the moments and forces that result from these waves and hence many accidents that happen on the sea can be avoided.

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