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Research Paper

Analyzing Tunisia's exchange rate fluctuations: Fuzzy filter approach

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Abstract. The standard approach analyzing the business cycle (RBC) is based on Hodrick-Prescott (HP) and Band-Pass (BP) filters to extract the cyclical component of the aggregate time series. By Monte Carlo experiments using Matlab program, we compared these traditional filters to a more recent Fuzzy Regression Filter. Our aim is to help the user to choose the appropriate filter in the appropriate time series. Through our application on the exchange rate (Dinar/U.S.$), the results show an improvement of the Fuzzy filter when the time series is non-stationary. In fact, through only the Fuzzy filter, (i) there no imposed restrictions on time series distribution; and (ii) the use of traditional filters lead to an evaluation bias of the exchange rate cyclical components. Additionally, when the implemented exchange policy is based on HP and/or BP filter, it then could be misled. Would it be necessary in a further research to review the stylized facts established by the standard RBC model by applying the Fuzzy filter as an alternative?

Keywords: Business cycle, Hodrick-Prescott and band-pass filters, fuzzy filter, FCM algorithm, Monte Carlo simulation, TDN/USD exchange rate.

INTRODUCTION

The instability of the exchange rate of the Tunisian Dinar against the Euro and the dollar observed in 2013 was due to several factors, mainly the recession in Europe and the nature of the controlled-flexible exchange rate regime adopted in the current Tunisian transition. One consequence of the instability of the national currency was the instability of the public spending which led to a revision of the budget choices at the end of this fiscal year. Indeed, the initially projected deficit of 5.9% in the budget was brought to 6.8% through additional regulatory measures and rationalization of public expenditure implemented by the authorities to avoid wider levels of deficit.[1] Macroeconomic balances are still fragile given the uncertainty of projected tax revenues and external funding. It seems that the forecasts made during the construction of the state budget as well as best practices for filtering time series, especially those on the exchange rate were not adopted. The application of filtering methods on macroeconomic time series allows obtaining their stylized facts and provides a relevant analysis framework as well as the deduction of appropriate measures of economic policies.

The aim of this paper is to provide the most appropriate filter for the treatment of bilateral exchange rate (Dinar/$) by conducting statistical comparison between the most common filters in the empirical literature namely HP, PB and Fuzzy filters. This statistical approach consists in Monte Carlo simulation of a time series with the trend and the cyclical components and comparing it with the filtered ones using the HP, BP and the Fuzzy filters.

[1] A complementary finance law, in January 2013. Official document. Minister of Finance. Tunisia.
http://www.tunisie.gov.tn/index.php?option=com_frontpage&Itemid=61

This paper is organized as follows: presentation of an overview of the HP, BP and the Fuzzy filters, as well as their features; followed by the different models used in our simulation, after which the results of this experiment are discussed. After that, an empirical application on the Tunisian exchange rate (Dinar/Dollar) for the period 2000:M1 to 2009:M6 is presented. Finally, the conclusion is given.

FILTERING PROCEDURES: A LITERATURE REVIEW

The analysis of business cycle behavior has devoted much attention to the decomposition methodology of a time series into trend and cyclical components. In the literature, this method is known as the modern approach. Those concerned focus only on the cyclical components. This way contrasts with the traditional (classical) one mainly based on the seminal work of Burns and Mitchel (1946). Accordingly the business cycle is a sequence of expansions and contractions in the output levels without referring to any type of preliminary detrending time series.

The importance of the modern approach outlined by Giles and Stroomer (2004) is concerned by two contexts. First, it lies in verifying the asymmetry in the business cycle (Neftçi, 1984; Sichel, 1989, 1993; Giles, 1997; Verbrugge, 1997; Razzak, 2001; Psaradakis and Sola, 2003). Second, the importance appears in the context of measuring potential output (Giorno et al., 1995; Scott, 2000; Haltmaier, 2001); Boz et al. (2011) and more recently Tastan H. (2013). In addition, getting stationary components is one the most important advantage of building a filter.

Several filters are suggested to isolate the trend components from the time series. The most known are the HP filter suggested by Hodrick and Prescott (1980, 1997), the BN filter developed by Beveridge and Nelson (1981) and the Band-pass filter proposed by Baxter and King (1995).[2] Guay and ST-Amant (2005) study the capacity of the HP and BP filters to provide a better approximation of Business cycle. They show that these filters give good results if the trend components dominate the cyclical ones with a slight improvement of the HP filter. In the applied macroeconomics, the HP filter is the most utilized one. It is widely used in applications, as well as several statistics programs as Matlab and SPlus take the HP filter as the default filter.

During the last years other filters were developed. Among the latter, the fuzzy filter developed by (Giles and Stroomer, 2004), and based the Fuzzy regression analyzed by Giles and Draeseke (2003), seems to be suitable to decompose macroeconomic time series in trend and cyclical components. Although the HP and the BP filters are widely used in the macroeconomic time series analysis, the Fuzzy regression is also widely used in applications and the statistic programs. We focus our paper on these three filters.

Several authors such as Zarnowitz and Ozyildirim (2006) do not prefer the technique of the macroeconomic time series filtering. In fact, they noted that the case of a defective detrending can generate significant errors. The analysis can indeed be simple and reliable when no adjustment of tendency is carried out.[3]

HP filter

The HP filter, suggested by Hodrick and Prescott (1980, 1997), aims to extract the permanent component (trend) from the time series using the Lagrangian multiplier. Thus, it considers that any time series is the sum of a smooth trend component, T_i , and a cyclical ones, C_i . So, any time series, according to Hodrick and Prescott, is presented by the following equation:

$$Y_i = T_i + C_i \tag{1}$$

The HP filter becomes to minimize the difference between the observed time series and its trend component with some restrictions on the second difference of these components. Using the Lagrangien multiplier, the trend component, T_i, is defined by the following optimization program:

$$\{T_i\}_{i=0}^{n+1} = \arg\min\left(\sum_{i=1}^{n}(Y_i - T_i)^2 + \lambda\sum_{i=2}^{n-1}\left[(T_{i+1} - T_i) - (T_i - T_{i-1})\right]^2\right), \tag{2}$$

Where, n , and, λ , represent, respectively, the sample size and an integer number representing the smoothness parameter of the trend components. The latter depends upon the periodicity of the data. Hodrick and Prescott propose that the parameter is equal to 100, 1600, and 14400 if the data are respectively yearly, quarterly, and monthly.[4] Thus, the cyclical components, C_i, are gotten by isolating the trend components from the real time series.

In the last two decades several works focused on the evaluation of the HP filter as procedure extracting cyclical components.[5] Cogley and Nason (1995) studied the capacity of this filter to generate economic cycles for data

[2] Among the filters available in the literature, we can cite the example of Corbae and Ouliaris (2006), Harvey et al. (2004) and Berardi et al. (2011).

[3] Harvey (1993) shows that the uncritical use of mechanical detrending can lead investigators to report spurious cyclical behavior.

[4] More recently, Schlicht (2004) proposed the smoothness parameter estimation.

[5] See Maravall and del Rio (2001) for a survey.

with a stationary trend. This is illustrated as follows,
Let,

$y_t = \alpha + \beta t + z_t$, where z_t is a stationary and mixing;
The Difference-stationary processes is,

$\Delta y_t = \alpha + v_t$ with v_t is stationary mixing.

These authors show that the HP filter is able to produce a cyclical behavior of the series while in reality we do not observe important transitory fluctuations in the original data. Gay and ST-Amant (2005) revealed that this filter yields a false image in relation to the reality if the series spectrum of the filtered data is prevailed by the low frequencies.

However, Singleton (1988) highlights that for large sized samples, the HP filter is a good approximation of the high band-pass one. Through this transformation, the HP filter eliminates only the low frequency components (trend). This is the Baxter and King Criticism who developed the BP filter.

Band-pass filter

The Band-Pass filter (BP or BK) is proposed by Baxter and King (1995) which was extended later on by Christiano and Fitzgerald (1999). The idea is based on the definition of the business cycle suggested by Burns and Mitchel (1946) who define the business cycle as the fluctuations of the time series level in a specified range of periodicity. Baxter and King lean on the works achieved by the NBER: National Bureau of Economic Research, where they noticed that spurious American time series exhibit the cycles admitting a period understood between 6 and 32 quarters.[6] Consequently, their innovation "BP filter" extracts the components which have a periodic fluctuation between 6 and 32 quarters.[7] These components represent the filtered cyclical series. The trend components admit a period over 32 quarters and the components that admit a period lower 6 quarters is presented as irregular components.

Analytically, in order to define their filter, the authors followed a frequency approach. They define a frequency interval $[\omega_1, \omega_2]$ which keep the frequency belongs this interval and cancel the frequency lower than ω_1 and superior to ω_2. The procedure is based on the following transfer function:[8]

$$A(e^{-i\omega}) = \begin{cases} 1 & \text{if } \omega_1 \leq \omega \leq \omega_2 \\ 0 & \text{else} \end{cases} \quad (3)$$

Using the Fourier transformation, the transfer function takes the following expression:

$$A(e^{-i\omega}) = \sum_{k=-\infty}^{+\infty} a_k e^{-ik\omega} \quad (4)$$

Where the coefficients a_k are as follows

$$a_k = \frac{1}{2\pi} \int_{\omega_1}^{\omega_2} e^{ik\omega} d\omega$$

And,

$$a_0 = \frac{(\omega)}{2\pi} \quad \text{and} \quad a_h = \frac{\sin(\omega h)}{h\pi}$$

Using the BP method, the filtered series y_t^* is given by

$$y_t^* = \sum_{-\infty}^{+\infty} a_k y_{t-k} . \quad (5)$$

Baxter and King have defined the cyclical components as the part of the series whose period is consisted between two boundary-values, 6 and 32 quarter if the time series is quarterly, 2 and 8 if we have an annual time series, and 2 and 24 if we have a monthly time series.[9]

By our examination of the literature, we stressed the fact that a BP filter eliminates the frequencies that are out of a specific interval. The determination of this interval is based on the theoretical and applied works in the American economic context by admitting that the cycle durations range from 1.5 and 8 years. Thus the users of this filter have to accept these values. We consider that the extraction of cyclical component based on the fixed values presents a tremendous limit of the BP filter.

Fuzzy filter

The Fuzzy filter methodology based on the Fuzzy regression developed by Giles and Draeseke (2001, 2003) involved the clustering technique. The modeling procedure is based on the Fuzzy sets introduced by Zadeh (1995). It aims to cluster the data and attribute a membership grade of each data point in each cluster. The membership degree is a continuous number between zero and one where these limits values respectively indicate no membership and full membership.

[6] See Stock JH, Watson MW (1998) for the Business cycle fluctuations in U.S. macroeconomic time series.
[7] This approach is presented as the best filter for discriminating between the classical and the modern approach.
[8] See for example Brockwell and Davis, 1996, chapitre 4.
[9] See more details in Paul Cashin, 2004.

Giles and Draeseke (2001) summarize the modeling technique in three steps. The first is the clustering of the input data (explanatory variables) into fuzzy sets. The second is the estimation of the appropriate model of the explained variable over each set. The third is the combination of each sub-model into a single overall model by using the membership functions associated with the clustered data.

The first step of the modeling which consists in the clustering phase of the numerical data is solved by the Fuzzy C-means (FCM) clustering algorithm (Dunn, 1974; Bezdek, 1974). The FCM algorithm aims to partition the data into fuzzy sets and to quantify the degree of membership of every data-point with every cluster. A brief description of the algorithm is presented as follows.

The method used is a partitioning method that treats observations in the data as objects having locations and distances from each other. It partitions the data into "c" clusters, such that objects within each cluster are as close to each other as possible, and as far from objects in other clusters as possible. Each cluster is characterized by its center point. Various distances are identified to construct the cluster algorithm. In this paper, we use the Euclidean distance which is the default method in the clustering algorithm.[10]

The C-means algorithm known also as K-means algorithm has been widely applied. It is programmed in the Matlab and SPlus softwares.

More than the clustering phase, the FCM algorithm aims to identify the associated ''degree of membership", 'u_{ik}' for each data-point 'k' in cluster 'i'. It is based on minimization the following functional:

$$J(u,v)=\sum_{i=1}^{c}\sum_{k=1}^{n}(u_{ik})^m(d_{ik})^2 \qquad (6)$$

Where,

n is the sample sizes,

c is the number of clusters, and,

m is a constant such as $m \succ 1$.

Furthermore, $d_{ik}=\|x_k - v_i\|$ is the distance between the data, x_k, and the 'ith' cluster centre v_i, and u_{ik} is the degree of membership associated to each data-point, k, where, $\sum_{i=1}^{c} u_{ik}=1$.

The FCM algorithm requires introducing the cluster number c, and the fuzziness parameter, m. The common choice corresponding to the fuzziness parameter shows that $m=2$, but the cluster number can take $c=2, 3, \text{or } 4$ taking into account the sample sizes of the data that is going cluster as well as its distribution.[11] The steps of the algorithm are more developed in Giles and Sroomer (2006).

The membership value and the centers of the fuzzy clusters are obtained iteratively as follows:

$$u_{ik}=\left(\sum_{j=1}^{c}\left[(d_{ik})^2/(d_{jk})^2\right]^{1/(m-1)}\right)^{-1}$$

$$v_i=\left[\sum_{k=1}^{n}(u_{ik})^m x_k\right]\Big/\left[\sum_{k=1}^{n}(u_{ik})^m\right], \text{ for } i=1,2,...,c$$

For each cluster, we illustrate the fuzzy regression where we consider the case of a single regressed and a constant intercept (other than a no constant intercept).[12] The fuzzy relationship between the output and the regressor to be estimated is presented as follows:

$$y=f(x)+\varepsilon \qquad (7)$$

Where the functional relationship, $f(.)$, is unspecified and involves unknown parameters. A random disturbance term is presented by ε. No distributional assumptions need to be made about the latter. Giles's work suggests an additive model to the functional relationship and recommends the OLS method in order to estimate the unknown parameters.

The extraction of the cyclical components using the filter requires the identification and estimation of the fuzzy model over each cluster separately. Thus, according to the various works of Giles with several others, the three steps are presented as follows:

a) Partition the sample observations for x into c clusters, using the FCM algorithm.

b) Fit the appropriate model separately over each fuzzy cluster:

[10] Note that the Minkowski Metric is expressed as $d_p(x_i,x_j)=\left(\sum_{k=1}^{d}\left|x_{i,k}-x_{j,k}\right|^p\right)^{1/p}$. The Euclidean distance between x_i and x_j is a special case where *p=2*. In addition there are no general theoretical guidelines for selecting a measure for any given application. Other distance measurements are possible to be used for a specific issues as the *Manhattan* metric, when *p*=1, or the Mahalanobis distance which is based on the covariance between the variables by which various patterns can be identified. It is a useful way of determining similarity of an unknown sample set to a known one. However, as usually, when variables are of same units, the Euclidean distance, that we consider sufficient for our application, is adopted as in this paper.

[11] Feng and Giles (2007) proposed a method to the clusters numbers selection.

[12] Giles and Stroomer (2006) generalize this case with multiple regressors.

$$y_{ij} = f_i(x_{ij}) + \varepsilon_{ij}, \quad j = 1,\ldots, n_i; \quad i = 1,\ldots,c.$$

Where, i and j represent respectively the cluster numbers and the sample sizes for each cluster. In the particular case of an additive linear model, the expression of the fuzzy model is as follows,

$$y_{ij} = \beta_0 + \beta_{i1}x_{ij} + \varepsilon_{ij}$$

c) Predict the conditional mean of the time series, y, using the following expression:

$$\hat{y}_k = \frac{\left[\sum_{i=1}^{c}(\hat{\beta}_{i0} + \hat{\beta}_{i1}x_k)u_{ik}\right]}{\sum_{i}^{c} u_{ik}} \quad (8)$$

Where,

u_{ik}, is the degree of membership of the k^{th} value of the explanatory variable, x, in the i^{th} fuzzy cluster. $\hat{\beta}_{i0}$ and $\hat{\beta}_{i1}$ are the least squares estimators. In this case, the fuzzy predictor, $\hat{y}_k$, (of the conditional mean of the time series, y) is a weighted average of the linear predictors based on the fuzzy partitioning of the explanatory data, with the weights (membership values) varying continuously through the sample (Giles and Draeseke, 2004). Indeed, the calculated predicted "input-output relationship" (that is, the derivative) between *x* and the conditional mean of *y* is as follows:

$$\frac{\partial \hat{y}_k}{\partial x_{ij}} = \left[\sum_{i=1}^{c}\left(\hat{\beta}_{i1}u_{ik}\right)\right] / \left[\sum_{i=1}^{c} u_{ik}\right], \qquad k = 1,\ldots, n \quad (9)$$

Giles and Stroomer (2004) noticed that this function (Equation 9) enables non-linearity to be effectively modeled and it provides a competitor for other filters such as the HP filter.

SIMULATION METHODOLOGY

We are going to focus on the steps of Guay and ST-Amant (2005) and Giles and Stroomer (2004) to simulate a data-generating process taking into account the trend and the cyclical components. Our experiment involves the permanent, T_t , and the cyclical, C_t, components of Equation 1 are respectively expressed as follows:

$$T_t = \alpha T_{t-1} + \varepsilon_t, \quad \text{and} \quad C_t = \phi_1 C_{t-1} + \phi_2 C_{t-2} + \eta_t$$

Where,

$$\varepsilon_t \sim NID(0, \sigma_\varepsilon^2), \quad \eta_t \sim NID(0, \sigma_\eta^2).$$

The series, Y_t, defined as the sum of a permanent component, T_t, which corresponds to a random walk, and a cylical component, C_t, follows a second-order autoregressive process.

Guay and ST-Amant (2005) dicussed the choice of the cyclical components dynamics as a second-order autoregressive process because its spectrum may have a peak at zero frequency or at business-cycle frequencies. The Gaussian error terms of the two components presented by, ε_t, and, η_t, are supposed uncorrelated.

For stationarity, some restrictions are imposed on the cyclical component expression. They concern the autoregressive parameters. Following Cryer and Chan (2008) the necessary and sufficient stationarity conditions in this $AR(2)$ process case are $\phi_1 + \phi_2 < 1, |\phi_2| < 1$, and $\phi_2 - \phi_1 < 1$. Indeed, for stationarity we require that the roots of quadratic characteristic polynomial, $\frac{\left[\phi_1 \pm (\phi_1^2 + 4\phi_2)^{1/2}\right]}{-2\phi_2}$, exceed 1. Some simple algebraic manipulations done by the authors lead to the stationarity conditions. [13]

Two cases are respectively studied in the works of Guay and ST-Amant (2005) and Giles and Stroomer (2004). The former have retained the hypothesis of a non-stationary stochastic trend (that is, $\alpha = 1$ as in Equation 1). The latter kept the hypothesis of a deterministic stationary trend (that is, $|\alpha| < 1$). In order to incorporate an extended data-generating process which takes into account more than the specific cases above, we adopt the model analyzed by Zivot and Wang (2002). Here, the trend process is as follows:

$$T_t = \alpha_1 t + \alpha_2 T_{t-1} + \varepsilon_t, \quad (10)$$

Where,

$|\alpha_2| < 1$, this trend is stationary, $AR(1)$ since it is $I(0)$ with a deterministic temporal trend (Zivot, Wang (2002). ε, the random disturbance term takes the same properties as in the above equation.

Various values of autoregressive parameters have been studied for the cycle and the trend expressions and for different ratios of the error variance. The sample size

[13] The proof is available in Cryer and Chan (2008), P.84.

Table 1. Monte Carlo Simulation: Stochastic trend model (α = 0.1).

Parameters			Correlation			Volatility				
$\frac{\sigma_\varepsilon}{\sigma_\eta}$	ϕ_1	ϕ_2	HP	BP	Fuzzy	HP	BP	Fuzzy	True	Time series
	1.3	-0.35	0.768 (0.000)	0.765 (0.000)	0.854 (0.000)	108.065	198.92	154.55	198.92	200.29
0.01	1.3	-0.65	0.913 (0.000)	0.858 (0.000)	0.990 (0.000)	147.65	150.89	172.69	175.63	176.01
	1.3	-0.85	0.964 (0.000)	0.883 (0.000)	0.991 (0.000)	205.41	193.36	217.03	220.61	220.19
	1.3	-0.35	0.76 (0.000)	0.763 (0.000)	0.845 (0.000)	15.49	10.87	15.75	19.89	21.49
0.1	1.3	-0.65	0.948 (0.000)	0.942 (0.000)	0.982 (0.000)	18.76	20.48	21.08	21.77	22.37
	1.3	-0.85	0.946 (0.000)	0.937 (0.000)	0. 981 (0.000)	19.41	20.07	21.1	21.83	21.62
	1.3	-0.35	0.178 (0.077)	0.282 (0.005)	0.755 (0.000)	10.34	15.45	32.97	12.46	39.15
1	1.3	-0.65	0.531 (0.000)	0.456 (0.000)	0.409 (0.000)	11.26	15.23	27.17	8.73	34.83
	1.3	-0.85	0.326 (0.001)	0.697 (0.000)	0.567 (0.000)	13.31	16.24	26.54	8.12	35.29
	1.3	-0.35	0.347 (0.000)	0.292 (0.003)	0.049 (0.627)	4.78	5.87	10.18	2.00	16.86
5	1.3	-0.65	0.337 (0.000)	0.368 (0.000)	0.211 (0.035)	4.56	5.98	10.51	2.23	16.87
	1.3	-0.85	0.573 (0.000)	0.488 (0.000)	0.277 (0.05)	5.25	6.33	10.65	3.22	17.10
	1.3	-0.35	-0.346 (0.000)	-0056. (0.578)	0.026 (0.799)	9.16	13.19	22.88	4.77	33.53
10	1.3	-0.65	0.326 (0.000)	0.451 (0.000)	0.29 (0.003)	9.83	13.99	25.31	1.94	34.42
	1.3	-0.85	0.154 (0.127)	0.029 (0.772)	0.078 (0.442)	9.68	13.07	24.98	2.56	33.71

(.): p-value in parenthesis

considered is $n=100$, for every simulation and the number of repetitions is 1000. The smoothing parameter for the HP filter was set to 100, the interval in the BP filter is [6, 32], and the number of clusters is $c=2$ for the fuzzy filter.

SIMULATION RESULTS

There are several ways to establish comparisons between different filters. We compare the correlation and the volatility between the true cyclical components and the extracted cyclical ones using alternatively the Fuzzy, HP, and the BP filters. This procedure shows the advantageous filter and allows studying its performance to produce adequate cyclical components of the true cyclical ones for different models that the macroeconomic time series can follow. Giles and Stroomer (2004) noted that the first and the last eight filtered data-points provide a poor performance of the HP filter at the ends of the sample owing to its weakness at the level of extreme points as previously stated. In this paper, we have kept all the data-points in the two filters to maintain the series characteristics.

As shown in Tables 1, 2 and 3 of the results of Monte-Carlo simulations according to various values of the ratio $\sigma_\varepsilon / \sigma_\eta$, and for various autoregressive parameters ϕ_1 and ϕ_2, show that for low values of these ratios (indicating a performance of the filter) the Fuzzy filter is the best. This is the case of a stochastic trend ($\alpha = 1$), or deterministic trend ($|\alpha| \prec 1$). Indeed, it is clear that for low values of this ratio, that is, 0.01; 0.1, the correlation is statistically significantly closer to the unit for the Fuzzy filter than for the HP and BP filters (p-values are in parentheses). This same result of performance of the Filter compared to HP and BP filters is obtained in the case of the extended generating data process we considered, as shown in Table 3 of the of the Monte-Carlo simulation results. Thus, through our simulations, the Fuzzy filter allows to obtain more powerful results than those obtained by HP and BP filters when the cyclical component is dominant, that is $\sigma_\varepsilon / \sigma_\eta \leq 1$.

The advantage of the Fuzzy filter is that it does not involve any constraints, either in terms of a parametric specification as is the case with the HP filter, or in terms of fixing the duration, as is the case with the BP filter. The main result we conclude from the three tables is the well known performances of these filters when the transitory components dominate the permanent ones. However, when the trend component is dominant $\sigma_\varepsilon / \sigma_\eta \succ 1$, the results show that the three filters do not seem to be suitable in the analysis of the cycles. In this case the

Table 2. Monte carlo simulation: deterministic trend model $(|\alpha|<1)$.

Parameters			Correlation			Volatility				
$\frac{\sigma_\varepsilon}{\sigma_\eta}$	ϕ_1	ϕ_2	HP	BP	FUZZY	HP	BP	FUZZY	True	Time series
	1.3	-0.35	0.768 (0.000)	0.765 (0.000)	0.855 (0.000)	108.11	155.88	154.54	198.92	198.70
0.01	1.3	-0.65	0.913 (0.000)	0.858 (0.000)	0.990 (0.000)	147.35	150.5	172.43	175.63	175.46
	1.3	-0.85	0.964 (0.000)	0.883 (0.000)	0.991 (0.000)	205.49	193.61	217.33	220.61	220.89
	1.3	-0.35	0.763 (0.000)	0.764 (0.000)	0.849 (0.000)	10.92	15.6	15.72	19.89	19.73
0.1	1.3	-0.65	0.949 (0.000)	0.942 (0.000)	0.984 (0.000)	18.56	20.31	20.91	21.77	21.56
	1.3	-0.85	0.948 (0.000)	0.939 (0.000)	0.983 (0.000)	19.74	20.23	21.42	21.83	22.21
	1.3	-0.35	0.143 (0.157)	0.312 (0.002)	0.764 (0.000)	10.45	12.67	21.33	12.46	24.28
1	1.3	-0.65	0.545 (0.000)	0.515 (0.000)	0.499 (0.000)	12.41	12.87	16.45	8.73	18.63
	1.3	-0.85	0.426 (0.000)	0.425 (0.000)	0.414 (0.000)	10.91	11.03	15.67	8.12	16.81
	1.3	-0.35	0.246 (0.014)	0.367 (0.000)	0.367 (0.000)	4.59	4.82	6.57	2.00	7.73
5	1.3	-0.65	0.138 (0.169)	0.192 (0.056)	0.132 (0.191)	4.28	4.35	6.07	2.22	7.35
	1.3	-0.85	0.529 (0.000)	0.508 (0.000)	0.417 (0.000)	5.23	5.25	6.79	3.22	7.86
	1.3	-0.35	0.077 (0.445)	0.085 (0.399)	-0.076 (0.452)	8.50	8.80	11.68	4.77	13.43
10	1.3	-0.65	0.263 (0.008)	0.106 (0.295)	0.188 (0.061)	8.72	8.56	13.13	1.94	15.18
	1.3	-0.85	0.094 (0.354)	0.187 (0.063)	0.103 (0.306)	8.48	8.44	12.92	2.56	15.13

(.): p-value in parenthesis.

coefficients of correlation are low and are around 0.3 for the Fuzzy filter, 0.2 for the HP filter and 0.1 for the BP filter. These results remain verified in models with stationary or stochastic trend.

In practice the HP filter imposes some restrictions on the parameter of smoothness, according to whether the data are monthly $(\lambda=14400)$, quarterly $(\lambda=1600)$ or annual $(\lambda=100)$. The BP filter imposes also restriction on the data frequency, while the Fuzzy filter does not impose any constraint. Nevertheless, the common constraint for the all three filters is that of stationarity conditions of cyclical component when Monte-Carlo simulations are implemented. To distinguish between three studied cases, we choose $\alpha=1$ for a deterministic trend, $|\alpha|<1$ for a stochastic trend, and $T_t=\alpha_1 t+\alpha_2 T_{t-1}+\varepsilon_t$ for an extended generating-data process.

EMPIRICAL EVIDENCE

To apply these techniques, the HP, the BP and the Fuzzy filters, in order to extract the components cyclical of a Tunisian macroeconomic time series, we consider the monthly market exchange rate (Dinar/Dollar) on the 2000:M1 to 2009:M6, taken from the IFS (2009) database.[14] Drawn in Figure 1.

The cyclical components of the exchange rate time series extracted by these filters are made into just one (Figure 2) in order to easily study the distinction between one and other. Figure 2 shows similarity between the Fuzzy and the HP cyclical components. The BP filter exhibits cyclical components which are more volatile than the others. It's recommended that the Fuzzy and the HP filters, rather than the BP one, should be used to analyze the fluctuations of the exchange rate. The analysis of the growth cycles, illustrated by these different filters, is done by the dating procedure using the Bry and Boschan (1971) algorithm. The detailed results are presented in Table 4 which shows that the Fuzzy cyclical component exhibits three peaks and three troughs. The BP cyclical component exhibits little more turning points (four peaks and four troughs), and the HP filter shows only two peaks and two troughs. The results of the dating are drawn in Figures 3, and 4, and 5 where the shaded areas present the expansionary periods. Since the results obtained by the Fuzzy filter are located half-way, it makes us think that its results are more appropriate than those from the

[14] Borio (2013) notes the importance of the financial cycle to understand business fluctuations and the corresponding analytical and policy challenges.

Table 3. Monte Carlo simulation. Extended data-generating process $T_t = \alpha_1 t + \alpha_2 T_{t-1} + \varepsilon_t$,

Parameters			**Correlation**			**Volatility**				
$\frac{\sigma_\varepsilon}{\sigma_\eta}$	ϕ_1	ϕ_2	**HP**	**BP**	**Fuzzy**	**HP**	**BP**	**Fuzzy**	**True**	**Time series**
	1.3	-0.35	0.406 (0.000)	0.522 (0.000)	0.950 (0.000)	9.67	16.76	36.85	39.4	41.40
0.01	1.3	-0.65	0.931 (0.000)	0.918 (0.000)	0.966 (0.000)	22.85	25.38	26.97	27.62	29.77
	1.3	-0.85	0.985 (0.000)	0.966 (0.000)	0.985 (0.000)	25.80	26.67	27.47	27.69	28.78
	1.3	-0.35	0.309 (0.002)	0.478 (0.000)	0.885 (0.000).	12.09	18.45	46.64	39.4	63.47
0.1	1.3	-0.65	0.858 (0.000)	0.857 (0.000)	0.801 (0.000)	23.16	24.71	32.60	27.62	49.22
	1.3	-0.85	0.931 (0.000)	0.910 (0.000)	0.791 (0.000)	23.81	24.38	30.15	27.69	44.28
	1.3	-0.35	0.311 (0.002)	0.487 (0.000)	0.825 (0.000)	13.97	21.77	46.81	39.40	57.72
1	1.3	-0.65	0.872 (0.000)	0.872 (0.000)	0.759 (0.000)	24.78	27.79	37.4	27.62	46.81
	1.3	-0.85	0.931 (0.000)	0.925 (0.000)	0.779 (0.000)	28.71	29.20	37.69	27.69	44.7
	1.3	-0.35	0.250 (0.012)	0.329 (0.001)	0.089 (0.377)	4.25	4.44	15.41	2.00	33.48
5	1.3	-0.65	0.464 (0.000)	0.549 (0.000)	0.108 (0.285)	4.56	4.98	15.43	2.22	33.81
	1.3	-0.85	0.581 (0.368)	0.571 (0.333)	0.172 (0.087)	4.95	5.03	15.58	3.22	138.61
	1.3	-0.35	0.099 (0.326)	-0.006 (0.952)	-0.245 (0.014)	8.72	9.93	71.53	2.27	137.41
10	1.3	-0.65	0.053 (0.598)	-0.009 (0.925)	-0.055 (0.589)	8.50	9.92	71.97	1.86	138.61
	1.3	-0.85	0.148 (0.143)	0.084 (0.408)	0.003 (0.978)	8.58	10.03	72.07	2.79	138.71

(.): p-value in parenthesis

Table 4. Chronology of the growth cycles of the exchange rate TDN/USD.

	Fuzzy filter	**HP filter**	**BP filter**
Peak	2002-1		
Trough			
Peak		2002-3	2002-3
Trough	2003-5		2003-5
Peak			
Trough		2003-12	
Peak			2004-4
Trough			
Peak	2004-7		
Trough	2004-12		2004-12
Peak		2005-11	2005-11
Trough			
Peak	2005-12		
Trough			2006-5
Peak			2007-7
Trough	2008-3	2008-3	
Peak			
Trough			2008-7

Table 5. Characteristic of the recession cycles (P-P).

Cycle	Fuzzy filter	HP filter	BP filter
Cycle 1	30 (221.6%) [33.23]	44 (186.9%) [41.12]	25 (211%) [26.38]
Cycle 2	17 (218.6%) [18.6]		19 (253.8%) [24.11]
Cycle 3			20 (172%) [17.2]

…: Length. (…): Depth […]: severity. The depth and severity are measured as follows:

$$Depth = \begin{cases} (y_p - y_c)/y_p : \text{ recessioncycle} \\ (y_c - y_p)/y_c : \text{ expansion cycle} \end{cases}$$

$$Severity = 0.5 * Depth * length$$

Table 6. Characteristic of the expansion cycles (C-C).

Cycle	Fuzzy filter	HP filter	BP filter
Cycle 1	19 (87.4%) [8.3]	51 (157.9%) [40.25]	19 (146.1%) [13.88]
Cycle 2	39 (162.6%) [31.71]		17 (254%) [21.6]
Cycle 3			26 (172.1%) [22.37]

…: length. (…): Depth. […]: severity

Table 7. Dating the growth cycles using Fuzzy, HP and BP filters.

Parameter	Fuzzy filter	HP filter	BP filter
Number of peaks	3	2	4
Number of troughs	3	2	4
Number of expansion phases	2	1	3
Number of recession phases	3	2	4
Number of expansion cycles (C-C)	2	1	3
Number of recession cycles (P-P)	2	1	3

other filters considered. Although figure 2 shows similarity between Fuzzy and HP filters, the dating of the turning points displays a coincidence in only one trough according to Table 4, we notice only one coincidence in March 2008. However, there is a coincidence in two troughs between the Fuzzy and the BP filters, while there are two common peaks between the HP and the BP filters. Moreover, the results in Table 4 and 7 show a steady rhythm in the majority of the expansions and recessions of cycles using Fuzzy and HP filters although the differentiation at the level of the turning points.

In order to analyze the growth cycles of the exchange rate further, we have analyzed each cycle separately by determining the depth and the severity of the expansion and recession cycles. The results are summarized in Tables 5 and 6. Note that an increase in foreign exchange rate means depreciation of the Dinar compared to the Dollar. The results of Tables 5 and 6 have important consequences in terms of macro-economic policies. It is important that the decision maker correctly identifies the cyclical behavior of foreign exchange rate. Based on the filter HP, we would conclude that foreign exchange rate is characterized by only one cycle of recession and is appreciated at the rate of 186%. With the same filter, the exchange rate is also characterized by only one expansion cycle and is depreciated at the rate of 157.9%. It is then possible that it implements corresponding macroeconomic policy according to its objective. If the objective is to maintain a competitiveness of the exports, the decision maker will reinforce this depreciation within sight of the current balance equilibrium.

However, with the Fuzzy filter recommended by our investigations, they are two cycles of recession whose appreciation is of approximately at the rate of 220% in each cycle. This could exceed target foreign exchange rate and bring to the consequences of the overvaluation of foreign exchange rate. As for the cycles of expansion, the depreciation of foreign exchange rate is around 87.7% in the first cycle and of 162.6% in the second cycle. This should certainly correspond to other macroeconomic policies. Based on a not-powerful filter in

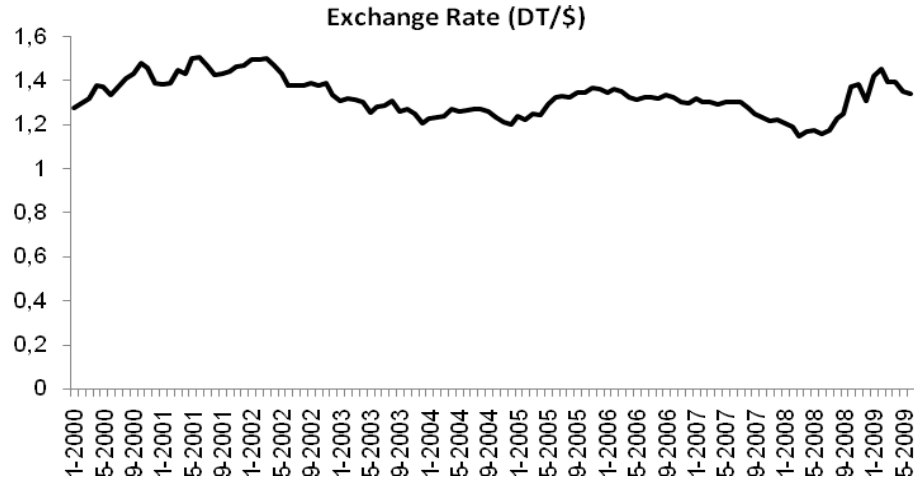

Figure 1. Exchange Rate series (DT/$): 2000:2009.

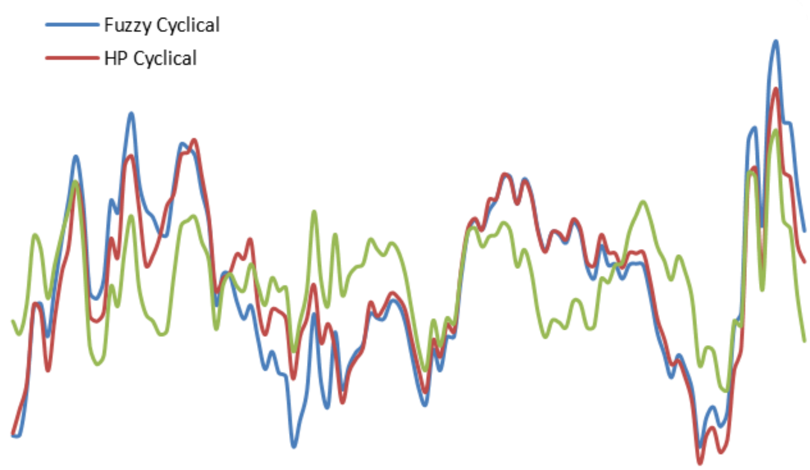

Figure 2. Extracting cyclical components of the Market Rate, End of Period using three filters.

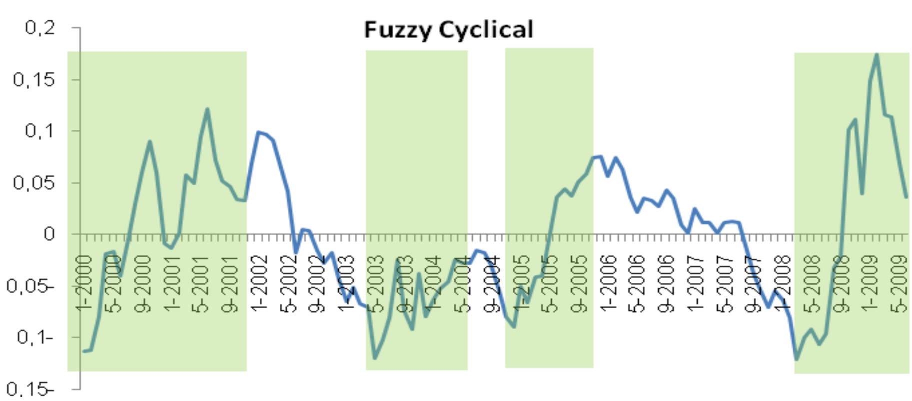

Figure 3. Dating the expansion phases of the Fuzzy Cyclical Components of the exchange rate.

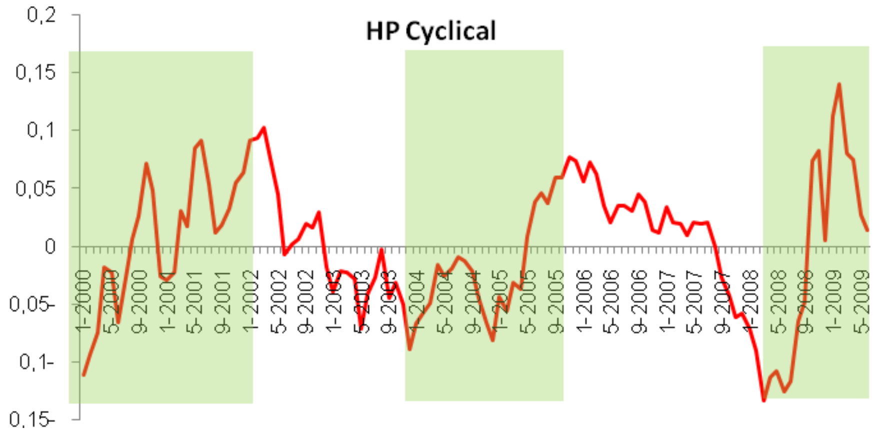

Figure 4. Dating the expansion phases of the HP Cyclical Components of the exchange rate.

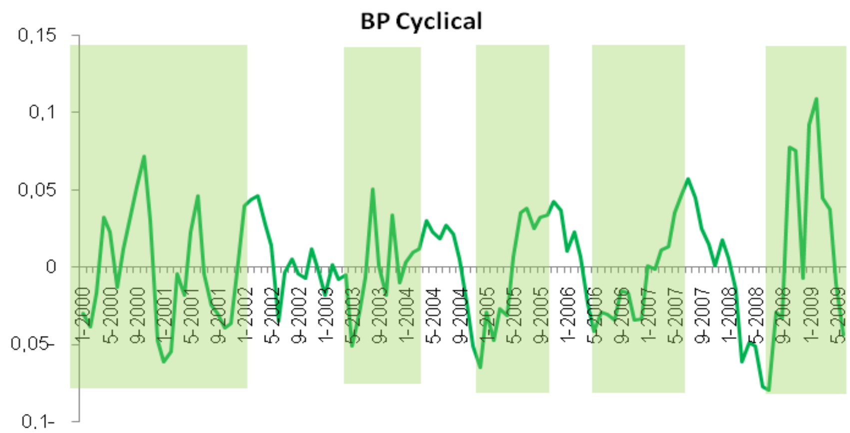

Figure 5. Dating the expansion phases of the BP Cyclical Components of the exchange rate.

the identification of the cyclical fluctuations of foreign exchange rate could induce the decision maker in error at the moment of the implementation of macro-economic policies and those relating to the exchange rate regime.

CONCLUSION

By Monte-Carlo simulations, we showed that the Fuzzy filter is the most appropriate detrending method compared to both HP and BP filters when the time series, that is going to be filtered, is characterized by a domination of the transitory components on the permanent ones. Moreover, contrary to the two other filters, the Fuzzy filter does not impose any parametrical restrictions for stationary cyclical component. Our application of the three filters on the Tunisian exchange rate (Dinar/Dollar) for the period 2000:M1 to 2009:M6 shows that when the decision maker is based on a not-powerful filter, that is, HP or BP filter, rather than Fuzzy one, he/she could induce in error and implement inappropriate exchange policies regimes.

The Fuzzy filter we propose for extracting cyclical macroeconomic time series is not widely used in the applied macro-econometric literature. The majority of works are based on the traditional filters in which we demonstrated the insufficiencies. These filters are adopted by the pioneer models of the RBC having influenced the literature during the three last decades (Kydland and Prescott, 1982, 1990; McGrattan, 1994; Chari et al., 2004). It would be then very important to revisit these models via the Fuzzy filter and to possibly establish other types of stylized facts.

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